Relative locality and relative Co-locality as extensions of the Generalized Uncertainty Principle

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Abstract

We interpret Relative locality as the variation of the Ultraviolet (UV) cut-off with respect to the observer's position relative to an event in agreement with an extended version of the Generalized Uncertainty Principle (GUP). As a consequence there is a natural red-shift effect for the events when they are observed at a given distance x. We then introduce the concept of Relative Co-locality as the variation of the infrared (IR) cut-off with respect to the observer's momentum relative to the event. As a consequence, there is a natural blue-shift effect for the events when the observer has a given momentum p with respect to them. Both effects are dual each other inside the formalism of quantum groups $SU(n)_q$ symmetric Heisenberg algebras and their q-Bargmann Fock representations. When Relative locality and Co-locality are introduced, the q-deformation parameter takes the form $q \approx 1 + \sqrt{\frac{|p||x|}{r_\Lambda m_{pl}c}}$ with the Relative Co-locality defined as $\Delta P \approx \frac{|p|}{r_\Lambda}\Delta X$, where $r_\Lambda = \frac{1}{\sqrt{\Lambda}}$ is the scale defined by the Cosmological Constant Λ, ΔX is a scale of position or time associated with the event, p and x are the momentum and position of the observer relative to the event.

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I. INTRODUCTION

Every approach of Quantum Gravity agrees in the fact that the spacetime becomes discrete at the Planck scale. It has been recently argued that it is not necessary to go to the Planck scale in order to perceive the effects of Quantum Gravity [1–3]. In fact, the Gamma ray bursts and the possibility of observing some energy dependent delays of arrival times of photons produced simultaneously at a given distance offers a good possibility for testing the geometry of momentum space. This is the so-called principle of Relative Locality introduced by Giovanni Amelino-Camelia, Laurent Freidel, Jerzy Kowalski-Glikman and Lee Smolin [2, 3]. In such a case, the Born principle already formulated in 1.938 is taken seriously [3]. Relative locality is an extension of the Relativity principle to the phase space, this principle suggests that events that are local with respect to one observer, are not necessarily local with respect to an observer at a different position. The observers basically live in different spacetimes created as the cotangent bundle of the momentum space [4]. The fundamental scale of Relative Locality is the Planck mass m_{pl}. The effects of Relative locality can then be observed even at astrophysical scales. The regime of Relative locality is $G_N \to 0$ and $\hbar \to 0$ but keeping $m_{pl} = \sqrt{\frac{\hbar}{G_N}}$ constant. Still is open the question of how can we include the spacetime curvature effects inside this formalism and how can be introduced the Cosmological Constant Λ scale. There is a proposal due to Giovanni Amelino-Camelia, Antonio Marciano, Marco Tanassa and Giacomo Rosati [5]. Here we take an alternative path by extending the notions of quantum groups $SU(n)_q$ symmetric Heisenberg algebras and their q-Bargmann Fock representations. We postulate the concept of Relative co-locality as a dual version of the Relative Locality principle. This is one way for introducing the curvature effects inside the Relative locality principle. The expression for Relative co-locality is $\Delta P \approx \frac{|p|}{r_\Lambda}\Delta X$, where ΔX could in principle be taken as a scale of position or time related to the event. The Relative Co-locality expression is just the dual version of the Relative locality principle which respects the $SU_q(n)$ symmetry with a q-deformation parameter $q \approx 1 + \sqrt{\frac{|p||x|}{r_\Lambda m_{pl} c}}$. This is just an extension of the deformation parameter q when we take l_{pl} as the minimum scale in position and $\frac{1}{r_\Lambda}$ as the minimum scale in momentum [6]. Relative Locality can be interpreted in 3 ways. 1). As the increase of the non-locality with respect to the distance to the event. 2). As a red-shift effect with respect to the distance to the event. 3). As a variation of the effective Planck scale with respect to distant observers in agreement with the extension of a Generalized Uncertainty Principle (GUP). On the other hand, the Relative Co-locality can be interpreted in 3 ways: 1). As the increase of the non-locality of momentum space (uncertainty in momentum) with respect to the motion of an observer. 2). As a blue-shift effect with respect to the observer motion. 3). As a variation of the effective Cosmological Constant with respect to the motion. In a general process or event, Relative locality effects must compete with the Relative co-locality ones such that they cancel partially each other. The paper is organized as follows. In Section II, we introduce a new interpretation of the Relative Locality principle by introducing it as a variation of the UV cut-off scale with respect to the observer's position. In Section III, we make a review of GUP with UV and IR cut-offs inside the q-Bargmann Fock formalism already suggested by Kempf. In section IV, we introduce the principle of Relative Co-locality in order to introduce the Λ scale and restore the $SU_q(n)$ symmetry, necessary inside the q-Bargmann Fock formalism. Finally in Section VI, we conclude.

II. THE PRINCIPLE OF RELATIVE LOCALITY: A NEW INTERPRETATION

The principle of Relative Locality has already been established by Giovanni Amelino-Camelia, Laurent Freidel, Jerzy Kowalski-Glikman and Lee Smolin [2]. In such a case, the notion of locality depends of the observer's position with respect to the event. The events are (almost) local with respect to the observers near them, but non-local with respect to observers far away from it. This is a consequence of the fact that different observers construct different spacetimes as a cotangent bundle over a curved momentum space [2, 3]. Then if we have a collision at a given point in space for example, in principle an observer very far from the event will say that this collision was due to a non-local interaction or he/she will describe it as a non-local event. The non-local correction due to Relative Locality is given by [2, 3] ($c = 1$):

$$\Delta X \approx x \frac{E}{m_{pl}} \tag{1}$$

where E is the energy associated with the event; m_{pl} is the Planck mass and x is the relative position between the observer and the event. The Planck length is not the fundamental scale in Relative locality. The Relative Locality regime for example is $\hbar \to 0$, $G_N \to 0$ but keeping $m_{pl} = \sqrt{\frac{\hbar}{G_N}}$ fixed. Here we will show that the principle of Relative locality can be interpreted as an extension of GUP if the UV cut-off changes with respect to the observer's position. This is equivalent to a red-shift effect as we will explain in a moment. If we assume that the energy E in eq. 1 is related to the uncertainty in momentum of the event; then the Relative locality expression can be rewritten as:

$$\Delta X \approx |x| \frac{\Delta P}{m_{pl}c} \tag{2}$$

then if we write GUP with the UV cut-off as [6, 7]:

$$\Delta X \Delta P \geq \frac{1}{2} \left(\hbar + \frac{l_{pl}^2}{\hbar} (\Delta P)^2 \right) \tag{3}$$

eq. 2 could be interpreted as an additional uncertainty in position. If we multiply eq. 2 by ΔP, then we finally get:

$$\Delta X \Delta P \approx |x| \frac{(\Delta P)^2}{m_{pl}c} \tag{4}$$

if we compare with the UV cut-off for GUP given by 1, we observe that both expressions are comparable if:

$$l_{pl}^2 = |x| \frac{\hbar}{m_{pl}c} \qquad \to x = \frac{l_{pl}^2 m_{pl}c}{\hbar} \tag{5}$$

both expressions are the same as the observer is at a distance equivalent to the Planck length $x = l_{pl}$. Only an observer at this distance will perceive the events to be local. We can rewrite the GUP expression as:

$$\Delta X \Delta P \geq \frac{1}{2} \left(\hbar + \frac{|x|}{m_{pl}c} (\Delta P)^2 \right) \tag{6}$$

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in such a case, the UV cut-off is modified to $\Delta X_{min} = \sqrt{\frac{\hbar|x|}{m_{pl}c}}$ and $\Delta P_{UV} = \sqrt{\frac{\hbar m_{pl}c}{|x|}}$, the subindex UV means "Ultraviolet". The interpretation is then clear. An observer very far from the source, will perceive a larger minimum scale in position (increase of non-locality) and as a consequence, an smaller UV scale in momentum. The UV cut-off moves to the IR as the observer is far from the source. This is a natural red-shift effect and it is a consequence of Relative locality. In other words, an observer near the UV collision, will say that the cut-off is l_{pl}, on the other hand, an observer at a distance $|x|$, will say that the cut-off is moved to the IR in agreement with the expression 6. However, both observers would agree with a formulation of GUP in agreement with their experiences. They both live in different spaces, but in the same phase space. Let's assume now that the largest distance between the observer and the source is given by the Cosmological Constant Λ scale. For an observer at $r = r_\Lambda$, the GUP expression would be:

$$\Delta X \Delta P \geq \frac{1}{2}\left(\hbar + \frac{r_\Lambda}{m_{pl}c}(\Delta P)^2\right) \tag{7}$$

with the corresponding UV cut-off given by $\Delta X_{min} = \sqrt{l_{pl}r_\Lambda}$ and $\Delta P_{max} = \hbar\sqrt{\frac{1}{l_{pl}r_\Lambda}}$. This is an interesting result because this is the UV-IR mix scale already derived as a general extremal condition in [6] when l_{pl} is an UV cut-off and r_Λ is an IR one. In the present case, we see that this scale is the maximum degree of non-locality due to the Relative Locality effect. This is true if we assume that the maximum possible scale is given by Λ.

III. GUP WITH UV AND IR CUT-OFF SCALES: A REVIEW

In the next section we will introduce the Principle of Relative co-locality inspired in the notions of Relative locality and understanding it as an extension of the IR cut-off for GUP with a minimal scale in position and momentum. Before introducing the principle, let's first remember the derivation of GUP inside the q-Bargmann Fock formalism. In agreement with Kempf [8], we can define the Bargmann Fock operators:

$$\bar{\eta} := \frac{1}{2L}x - \frac{i}{2K}p \quad \partial_{\bar{\eta}} := \frac{1}{2L}x + \frac{i}{2K}p \tag{8}$$

where the constants L and K carry units of length and momentum. The commutation relations are then generalized to $\partial_{\bar{\eta}}\bar{\eta} - q\bar{\eta}\partial_{\bar{\eta}} = 1$; with $q \geq 1$. q is related to the gravitational degrees of freedom. This formalism has 2 free parameters with the constants K and L are related by $KL = \frac{1}{4}\hbar(q^2 + 1)$. The commutation relations of the position and momentum operator are now given by:

$$[x, p] = i\hbar + i\hbar(q^2 - 1)\left(\frac{x^2}{4L^2} + \frac{p^2}{4K^2}\right) \tag{9}$$

from this follows the uncertainty relation[8]:

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1 + (q^2 - 1)\left(\frac{(\Delta X)^2}{4L^2} + \frac{(\Delta P)^2}{4K^2}\right)\right) \tag{10}$$

here we have assumed $< X >=< P >= 0$ [6, 8]. In agreement with the formalism developed in [6, 8], the minimum scale in position corresponds to:

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$$\Delta X_{min} = L\sqrt{1 - q^{-2}} \tag{11}$$

additionally, it is known that the smallest uncertainty in momentum is given by:

$$\Delta P_{min} = K\sqrt{1 - q^{-2}} \tag{12}$$

K and L must satisfy the additional constraint:

$$KL = \frac{(q^2 + 1)\hbar}{4} \tag{13}$$

this condition suggests that the UV scale is dual to the IR one. For the moment, as a fact of illustration, the two free parameters will be fixed in agreement with the Planck scale l_{pl} and the Cosmological Constant scale $\Lambda = \frac{1}{r_\Lambda^2}$. We want an expression for GUP symmetric with respect to position and momentum given by:

$$\Delta X \Delta P \geq \frac{\hbar}{2} + \frac{l_{pl}^2}{2\hbar}(\Delta P)^2 + \frac{\hbar}{2r_\Lambda^2}(\Delta X)^2 \tag{14}$$

if we want this expression to agree with 10, the following conditions must be satisfied:

$$l_{pl} = L\sqrt{1 - q^{-2}} \tag{15}$$

$$\frac{1}{r_\Lambda} = K\sqrt{1 - q^{-2}}$$

these expressions together with the condition 13, automatically fix the value of q. Although q could take different values in order to satisfy the previous conditions, we select the value of q satisfying the additional constraint $q \geq 1$ [8]. In such a case, q, which is related to the gravitational degrees of freedom, satisfies:

$$q \approx 1 + \frac{l_{pl}}{r_\Lambda} \tag{16}$$

where $l_{pl} \approx 10^{-35}mt$ is the Planck scale and $r_\Lambda \approx 10^{26}mt$ is the Hubble one, then $q \approx 1 + 10^{-61}$. If $\Lambda \to 0$ then $q = 1$ and $l_{pl} \to 0$ and vice versa. It means that inside this formalism, the Cosmological Constant Λ is related to the minimum scale in position. Without a minimum scale, there is no Cosmological Constant and vice versa. Originally Kempf [6, 8] derived the results 11 and 12 by defining the function:

$$f(\Delta X, \Delta P) := \Delta X \Delta P - \frac{\hbar}{2}\left(1 + (q^2 - 1)\left(\frac{(\Delta X)^2 + <X>^2}{4L^2} + \frac{(\Delta P)^2 + <P>^2}{4K^2}\right)\right) \tag{17}$$

here, we will assume $<X> = 0 = <P>$. The minimum scale in position can be found given the following extremal condition:

$$\frac{\partial}{\partial \Delta P}f(\Delta X, \Delta P) = 0 \quad f(\Delta X, \Delta P) = 0 \tag{18}$$

then, the result is just the eq. 15 $\left(\Delta X_{min} = L\sqrt{1 - q^{-2}}\right)$. On the other hand, the minimum scale in momentum is obtained from the condition:

$$\frac{\partial}{\partial \Delta X} f(\Delta X, \Delta P) = 0 \quad f(\Delta X, \Delta P) = 0 \tag{19}$$

the result is just the eq. 15 $\left(\Delta P_{min} = K\sqrt{1 - q^{-2}}\right)$. We can however, derive a third scale given by the UV-IR mix effects. This scale was introduced for first time by John A. Wheeler [9] in 1957. It is given by the geometric average of the l_{pl} and r_Λ, namely, $l_0 = (l_{pl} r_\Lambda)^{1/2}$. We can define the total differential for the function $f(\Delta X, \Delta P)$ as:

$$df(\Delta X, \Delta P) = \left(\frac{\partial f(\Delta X, \Delta P)}{\partial \Delta P}\right)_{\Delta X = C} d(\Delta P) + \left(\frac{\partial f(\Delta X, \Delta P)}{\partial \Delta X}\right)_{\Delta P = C} d(\Delta X) \tag{20}$$

The general extremal condition inside the phase space is obtained as the total differential 20 goes to zero. In such a case:

$$df(\Delta X, \Delta P) = 0 \tag{21}$$

obtaining then the result:

$$\frac{d(\Delta X)}{d(\Delta P)} = -\frac{\Delta X}{\Delta P} \frac{\left(1 - \frac{\hbar}{4K^2}(q^2 - 1)\frac{\Delta P}{\Delta X}\right)}{\left(1 - \frac{\hbar}{4L^2}(q^2 - 1)\frac{\Delta X}{\Delta P}\right)} \tag{22}$$

imposing then the additional condition $\frac{d(\Delta X \Delta P)}{d \Delta X} = 0 = \frac{d(\Delta X \Delta P)}{d \Delta P}$. Then, we have to satisfy:

$$\frac{\Delta X}{\Delta P} = -\frac{d(\Delta X)}{d(\Delta P)} \tag{23}$$

introducing this result inside the right-hand side of 22, we obtain:

$$(q^2 - 1)\left(\frac{\Delta X}{L^2} + \frac{\Delta P}{K^2}\frac{d(\Delta P)}{d(\Delta X)}\right) = 0 \tag{24}$$

this equation has two solutions. The first one is not interesting for us, because it suggests $q = 1$ which is a trivial condition because in such a case $d(\Delta X \Delta P) = 0$ everywhere. Additionally, $q = 1$ corresponds to the standard Bosonic algebra in agreement with [2, 6, 8]. We then do not consider that case here. The interesting case is:

$$\left(\frac{\Delta X}{L^2} + \frac{\Delta P}{K^2}\frac{d(\Delta P)}{d(\Delta X)}\right) = 0 \tag{25}$$

which in combination with 23 gives:

$$\Delta X = \pm\frac{L}{K}\Delta P \tag{26}$$

if we compare the expressions 14 with the one obtained in 10, then it is simple to verify that $(\hbar = 1)$:

$$L = K^{-1} = \frac{\sqrt{2}}{2}(l_{pl} r_\Lambda)^{1/2} \tag{27}$$

then the condition 26 becomes:

$$\Delta X = \frac{1}{2}(l_{pl}r_\Lambda)\Delta P \tag{28}$$

here we have only taken into account the positive sign. For consistence, we can verify that the condition 13 is satisfied. If we take into account that in agreement with 16, we have $q^2 \approx 1 + 2\frac{l_{pl}}{r_\Lambda}$. Then 13 becomes ($\hbar = 1$):

$$KL \approx \frac{1}{2} \tag{29}$$

This result is consistent with 27 as can be verified. If we replace 28 inside 14, we then obtain under the approximation $r_\Lambda >> l_{pl}$, the following result:

$$\Delta X \approx (l_{pl}r_\Lambda)^{1/2} = l_0 \quad \Delta P \approx \frac{1}{(l_{pl}r_\Lambda)^{1/2}} = \frac{1}{l_0} \tag{30}$$

The result 30 is just the UV-IR scale already suggested by John A. Wheeler in 1957 [9] and interpreted as a coherence region. Note that this is also the maximum possible degree of non-locality in agreement with the expressions obtained after eq. 7. In such a case, the Λ scale was introduced artificially as the maximum possible distance for an observer relative to some event.

IV. THE PRINCIPLE OF RELATIVE CO-LOCALITY

Here we introduce the principle of Relative Co-locality inspired in the notions of Relative Locality and the IR cut-off in GUP. This is one way to introduce the notions of Relative locality in a curved spacetime. In this way, we make a full extension of the Born principle and additionally we introduce the Cosmological Constant scale. The expression which provides consistence with the $SU_q(n)$ symmetric formulation inside a q-Bargmann Fock formalism is:

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1 + \frac{|x|}{\hbar m_{pl}c}(\Delta P)^2 + \frac{|p|}{\hbar r_\Lambda}(\Delta X)^2\right) \tag{31}$$

where we have introduced the Relative Co-locality term as a correction of the IR cut-off term inside GUP. Note that the Relative Co-locality coefficient becomes the standard IR cut-off given in eq. 14 as $|p| = \frac{\hbar}{r_\Lambda}$, which is just the minimum momentum for the observer relative to the events. If we define the function:

$$f(\Delta X, \Delta P) := \Delta X \Delta P - \frac{\hbar}{2}\left(1 + \frac{|x|}{\hbar m_{pl}c}(\Delta P)^2 + \frac{|p|}{\hbar r_\Lambda}(\Delta X)^2\right) \tag{32}$$

then, we can find in analogy with 17 the minimal scales in position (UV cut-off) an momentum (IR cut-off) related to the concepts as they are described by an observer under the effects of Relative Locality and Co-locality. The minimal scale in position can be obtained with the conditions:

$$\frac{\partial f}{\partial \Delta P} = 0 \quad f(\Delta X, \Delta P) = 0 \tag{33}$$

then:

$$\Delta X_{min} = L\sqrt{\frac{q^2 - 1}{q^2}} \approx \sqrt{\frac{\hbar |x|}{m_{pl}c}} \tag{34}$$

on the other hand, the minimal scale in momentum can be obtained from the general condition 19 applied to 32. The result is:

$$\Delta P_{min} = K\sqrt{\frac{q^2 - 1}{q^2}} \approx \sqrt{\frac{\hbar |p|}{r_\Lambda}} \tag{35}$$

the results 34 and 35 are consistent with a q-Bargmann Fock algebra inside the $SU_q(n)$ in n dimensions, if we define the q-deformation parameter by:

$$q \approx 1 + \sqrt{\frac{|p||x|}{r_\Lambda m_{pl}c}} + \ldots \tag{36}$$

in agreement with the general constraint 13. Eq. 31 is consistent with the original GUP formulation 10 and the constraint 13. For deriving the result 36, we have assumed that the condition:

$$|x||p| << r_\Lambda m_{pl}c \tag{37}$$

is valid. Now we can derive the equivalent UV-IR mix scale when the effects of Relative locality and Relative co-locality are taken into account (together). The expression 26 is general. It represents the condition where the UV effects are equivalent to the IR ones. In the framework of Relative locality and Co-locality, the condition 26 is related to the scales at which the Relative locality effects cancel to the Relative Co-locality ones. We then need to obtain the new values for K and L already defined in 8 and 10. From 36 and 37, $q^2 \approx 1 + 2\sqrt{\frac{|x||p|}{r_\Lambda m_{pl}c}}$; we can then replace this result in the general expression 10:

$$\Delta X \Delta P \geq \frac{\hbar}{2}\left(1 + \frac{1}{2}\sqrt{\frac{|p||x|}{r_\Lambda m_{pl}c}}\left(\frac{(\Delta X)^2}{L^2} + \frac{(\Delta P)^2}{K^2}\right)\right) \tag{38}$$

comparing this result with the expression 31, we get:

$$K = \left(\frac{\hbar}{2}\right)^{1/2}\left(\frac{|p|}{|x|}\frac{m_{pl}c}{r_\Lambda}\right)^{1/4} \tag{39}$$

and:

$$L = \left(\frac{\hbar}{2}\right)^{1/2}\left(\frac{|x|}{|p|}\frac{r_\Lambda}{m_{pl}c}\right)^{1/4} \tag{40}$$

note that $KL \approx \frac{\hbar}{2}$; consistent with 13. If we replace these results in 26, we get:

$$\Delta X \approx \left(\frac{|x|}{|p|}\frac{r_\Lambda}{m_{pl}c}\right)^{1/2}\Delta P \tag{41}$$

this condition is obtained from the extremal condition 21 and 23 which are valid for any q-deformation parameter. Eq. 41 is the UV-IR mix condition under the Relative locality

and co-locality effects. If we replace 41 in 31, we then obtain the extended version of the UV-IR mix scale given by:

$$\Delta P_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|p|m_{pl}c}{|x|r_\Lambda}\right)^{1/4} \tag{42}$$

and:

$$\Delta X_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|x|r_\Lambda}{|p|m_{pl}c}\right)^{1/4} \tag{43}$$

the results 42 and 43 are just extensions of the results 30 but including the effects of Relative locality and Relative co-locality. Note that if $|p| = \frac{\hbar}{r_\Lambda}$ and $|x| = l_{pl}$, we recover the results 30 (with $\hbar = 1$). Only an observer at the Planck distance perceive the events to be local; and only an observer with a momentum given by the Λ scale, perceive the events to be Co-locals.

V. ISOLATED RELATIVE LOCALITY INSIDE THE $SU_q(n)$ DEFORMED HEISENBERG ALGEBRAS

The previous section just showed the most general case where the observer is both, very far from the event and He/She is also moving with respect to it. In this section, we want to recover the Relative locality (without Relative Co-locality) but inside the q-Bargmann Fock formalism. Here we will introduce the Λ scale as a fixed value. It is easy to demonstrate that we will recover the results of Section II if we impose the condition $r_\Lambda \to 1$.
If we want to isolate the Relative locality effects, the observer's momentum relative to the source must reach its minimum value given by $p = \frac{\hbar}{r_\Lambda}$; in such a case, the q-deformed parameter is just given by:

$$q \approx 1 + \frac{1}{r_\Lambda}\sqrt{\frac{\hbar|x|}{m_{pl}c}} \tag{44}$$

note that if $|x| = l_{pl}$, then we recover the results of Section III. Here however, we are interested in the Relative locality regime. All the relevant results are just extensions of those obtained in the previous section. The minimum scale in position for example given by eq. 34. However, the minimum scale in momentum is now given by the second equation of 15 (with $\hbar = 1$). The values of the scale parameters K and L are just extensions of 39 and 40 and they are given by:

$$K \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{\hbar m_{pl}c}{|x|r_\Lambda^2}\right)^{1/4} \tag{45}$$

and:

$$L \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|x|r_\Lambda^2}{\hbar m_{pl}c}\right)^{1/4} \tag{46}$$

additionally, the UV-IR mix scale is in this case, an extension of the results 42 and 43. The new UV-IR mix scales are:

$$\Delta P_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{\hbar m_{pl}c}{|x|r_\Lambda^2}\right)^{1/4} \tag{47}$$

$$\Delta X_{mix} \approx \left(\frac{\hbar}{2}\right)^{1/2} \left(\frac{|x|r_\Lambda^2}{\hbar m_{pl}c}\right)^{1/4} \tag{48}$$

The GUP expression in this case is:

$$\Delta X \Delta P \geq \frac{\hbar}{2} \left(1 + \frac{|x|}{\hbar m_{pl}c}(\Delta P)^2 + \frac{1}{r_\Lambda^2}(\Delta X)^2\right) \tag{49}$$

which is consistent with the results of the previous sections.

VI.   CONCLUSIONS

We have introduced a new interpretation of the principle of Relative Locality. We interpret it as a variation of the UV cut-off in a Generalized Uncertainty Principle inside the q-Bargmann Fock scenario. As a consequence of this, Relative locality produces a natural redshift effect for the observers located at a given distance relative to the event. If we try to localize a particle with a high energy photon for example at some point of the spacetime; for a distance observer this photon has a smaller frequency. For that observer, Relative locality produces a natural red-shift effect consistent with an extended version of GUP. On the other hand, if the same observer is moving with some momentum relative to the event (photon), there is an additional blue-shift effect produced by Relative Co-locality. We interpret the Relative co-locality as a variation of the IR cut-off in a Generalized Uncertainty Principle. If the observer is at some distance from the event and he/she is also moving with respect to it, then the Relative locality and co-locality effects compete each other. They become equally important at the extended UV-IR mix scales given by 42 and 43. Where ΔX and ΔP are the scales of position and momentum of the event and p with x are the scales of position and momentum of the observer relative to the event. In general, Relative locality can be interpreted as: 1). As the increase of the non-locality with respect to the distance to the event. 2). As a red-shift with respect to the distance to the event. 3). As a variation of the effective Planck scale with respect to distant observers in agreement with the extension of a Generalized Uncertainty Principle (GUP). Relative Co-locality can be interpreted as: 1). The increase of the non-locality of momentum space (uncertainty in momentum) with respect to the motion of an observer. 2). As a blue-shift effect with respect to the observer motion. 3). As a variation of the effective Cosmological Constant with respect to the motion.
Relative locality (alone) can be also introduced inside the q-Bargmann Fock formalism if the q-deformation parameter takes the value $q \approx 1 + \frac{1}{r_\Lambda}\sqrt{\frac{\hbar|x|}{m_{pl}c}}$. This is the case when the observer has a momentum relative to the source near to $|p| \approx \frac{\hbar}{r_\Lambda}$. In such a case, only the effects of Relative Locality can be perceived.

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